MODULE 1: THE DEVELOPMENT OF PLACE VALUE

Strange as it may seem we have never seen a number! More specifically, we have seen numbers as adjectives but not as nouns. For example, we have seen five people, five apples, five cows, five fingers; but we have never seen "five-ness" by itself.

Non-mathematical examples exist in abundance. For example, we can measure time to within a billionth of a second. Yet no one has ever seen time! What we see is the effect of what happens with the passing of time.

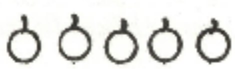
This is reflected in ancient sign languages such as the Egyptian hieroglyphics. If the picture representing a person was:

then

would represent five people.

And if the picture representing an apple was:

then



would represent five apples.

What we see are pictures of apples and people. There is no symbol that stands for "five".

It took people a long time to understand that mathematics was, in part, a language; and that, as in any language, symbols had to be invented to represent numbers.

> *Symbols used to stand for numbers are called* <u>numerals</u>.

This idea is explained in more detail in Lecture 0.

The first numerals were those that helped people *count*. In counting, a person would make one mark for each object that was being counted. The mark was usually a vertical straight line and it was called a *tally mark*.

For example, using tally marks we would represent five people, five apples, five fingers, or five horses simply by | | | | |.

When we write | | | | | five is still an adjective (modifying the number of tally marks) but it indicates that at last people understood that the meaning of "five" was the same whether we talked about people, apples, fingers, or horses. It had to be clear from the context what noun the tally marks were modifying.

For example, we talk about a herd of cows but a flock of sheep. Both "herd" and "flock" stand for "many", Yet we use different words because sheep and cows are different animals. But from a non-mathematical point of view, even though an orange tie and an orange shirt are different things, the adjective "orange" means the same thing in each case.

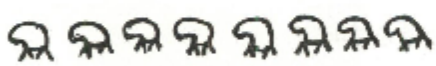
Example 1

Use tally marks to show how a shepherd might keep track of the fact that he had eight sheep.

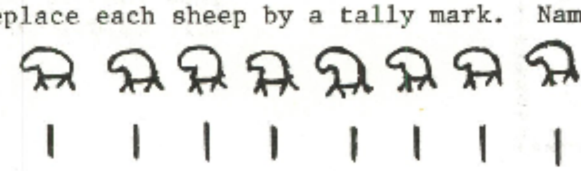
Answer: He'd write:
||||||||

(Notice that we'd have to know from context that the tally marks stood for sheep)

Had he been using hieroglyphics and if the symbol (picture) for a sheep was 

he'd write: 

Using tally marks he'd now simply replace each sheep by a tally mark. Namely:



Notice that it is much easier to draw tally marks than to draw sheep.

Of course, while tally marks were easy to understand, you can imagine how difficult it might have been to count them when we were dealing with large amounts.

Example 2

How would the shepherd have indicated by tally marks that he had twenty-four (24) sheep?

Answer: He'd write:

||||||||||||||||||||||||

The principle is exactly the same as in the previous example. He'd write one tally mark for each sheep. Since there were 24 sheep, he'd have written 24 tally marks.

The fact that we are endowed with ten fingers may have encouraged our shepherd in the last example to "bundle" the tally marks in groups of ten. For example, rather than write:

|||||||||||||||||||||||| (1)

he might have written:

|||||||||| |||||||||| |||| (2)

Notice that while lines (1) and (2) each have the same number of tally marks, line (2) seems easier to "count".

This same problem occurs even with smaller numbers of tally marks. Notice, for example, that on dice we do not use ''''' and '''''' to stand for five and six. Rather we use easy-to-recognize patterns:

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' '     ' '
 '  and ' '
' '     ' '
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This idea was further exploited by the ancient Egyptians and is described in Lecture 1.

As time went on, people realized that they could invent a special symbol to stand for ten. For example, the Romans used letters of their alphabet as numerals. Perhaps because of their resemblance, the letter I was used to stand for one tally mark.

That is, I and | resemble one another.

The Romans used the letter X to represent ten. While it is easy to memorize that X means ten or than XX means two X's, it is more important to realize that X in no way looks like it should mean ten. *In fact, it is quite likely that X would have stood for whatever number of fingers we happened to have!*

According to one account, the Romans would cross out tally marks in groups of ten. That is, . This was later abbreviated by using the "crossing-out" symbol without writing the tally marks. That is:

And the "crossing-out" symbol gave way to the X because of their resemblance.

The advent of place value did not depend on the fact that we have ten fingers. If we had been born with eight fingers, X most likely would have stood for eight.

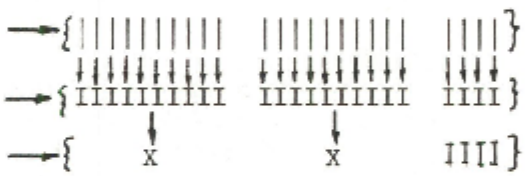
Once we know that X stands for ten it becomes easier to represent larger numbers.

Example 3

> Using X to stand for ten, how would the shepherd in Example 2 indicate that he had twenty four sheep?

Answer: He'd write XXIIII

Using I to stand for 1, he might have proceeded by the sequence of steps:



Using the symbols I and X no more than nine times each they could represent any number from one to ninety nine.

Example 4

> Using no more than nine each of the X's and I's, use Roman numerals to represent the number seventy five (75).

Answer: XXXXXXXIIIII

Seventy five means 7 tens and 5 ones.

That is, we need seven X's and five I's.

This would be written as XXXXXXXIIIII
1 2 3 4 5 6 7 1 2 3 4 5

(Eventually, as discussed in Lecture 1, the Romans used V for five and L for fifty; but for our purposes here, it is better to deal only with X and I.)

While XXXXXXXIIIII is much more difficult for us to read than 75, the fact is that it is still a marvelous improvement over having to write seventy-five separate tally marks.

The Romans then used the letter C to stand for ten X's (or equivalently, a hundred I's). They used the letter M (from the Latin word "millum" which means a thousand) to stand for ten C's or one thousand.

The Latin word for "hundred" is "centum". The C was probably chosen as an abbreviation for "centum" Notice the implication of a hundred in such words as "century", "cent", and "percent" (per hundred)

Roman Numerals
I means one
X means ten I's or ten
C means ten X's or a hundred
M means ten C's or a thousand

A system in which ten of a denomination is equal to one of the next higher denomination is called a decimal system. By omitting the symbols V(five), L(fifty), and D(five hundred), Roman Numerals are a decimal system.

Example 5

How would we express the number two thousand four hundred thirty one (2,431) using Roman numerals?

Answer: MMCCCCXXXI

We have 2 thousands or MM; 4 hundreds or CCCC; 3 tens or XXX, and 1 one or I.

Combining all the symbols we get:

MMCCCCXXXI

Again notice how 2, 3, 4, and 1 appear as adjectives while I, X, C, and M appear as the nouns they modify.

It is traditional to write Roman numerals from the greatest denomination to the least, but as mentioned in Lecture 1, this isn't at all crucial.

The numbers: one, ten, hundred, thousand, and so on are called *powers of ten*. To use Roman numerals correctly, you'd need a new numeral (letter of the alphabet) for each additional power of ten. In terms of today's technology, we'd have to use the whole

alphabet (and then some!) to write numbers that occur in today's world of science and technology.

For example, think of the number of miles between the earth and some distant stars in the galaxy; or even of the number of atoms in just an ounce of water.

To help overcome this problem, people came up with the idea of *place value*. In its earliest forms place value consisted of a series of vertical lines arranged in a horizontal row, *in which the lines represented the power of tens.*

In other words, the same symbol (a line) represented the various powers of ten. This replaced having to use a different-looking symbol (such as I, X, C, or M) for each power of ten.

As shown below, the line furthest to the left represented *ones*; the line immediately to its left represented the *tens*, and so on.

(thousands) *(hundreds)* *(tens)* *(ones)*

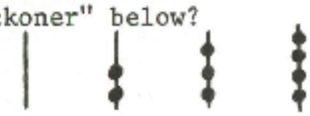
The spacing between the lines as well as the length of each line was not important. All that mattered is that the lines represented the powers of ten.

A marker placed on the ones-line meant 1 one; a marker placed on the tens-line meant 1 ten; a marker placed on the hundreds line meant 1 hundred; and so on. This device is still in wide use among Far Eastern people. It is called an *abacus*. The forerunner of the abacus was the *sand-reckoner*, in which the lines were drawn in the sand and the markers were stones or pebbles.

The Latin word for "stone" is "calculus". Hence to do arithmetic by using stones on lines meant to "calculate"

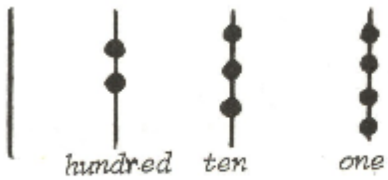
Example 6

What number is shown on the "sand-reckoner" below?



Answer: 234 (Two hundred thirty four)

Start by remembering the power of ten named by each line. Namely:



This immediately shows us that we have

2 *hundreds*, 3 *tens*, and 4 *ones*.

In this context, the lines represent the nouns (powers of ten) and the markers represent the adjectives. That is, the markers still behave like tally marks, but what they modify depends on which line they're on.

<u>*Note:*</u>

Observe that we know there are no thousands because of the absence of markers on the thousands-line. As long as the symbol for the power of ten is <u>visible</u>, we do not need an adjective to denote *none*. *This idea plays an important role in the next innovation.*

Using lines instead of letters of the alphabet was a big improvement; yet it was still awkward because of the great number of lines that would have to be used as the powers of ten increased. So the next innovation called for the invention of ten special numerals;

0, 1, 2, 3, 4, 5, 6, 7, 8, and 9

These numerals were called *digits*.

The word "digit" is Latin for "finger" or "toe". This serves as further evidence that the importance of ten is related to the fact that we have ten fingers.

Now instead of using special symbols like I, X, C, M or lines, we allow the *position* of the digit to determine the noun modified by the digit.

That is, just as in the sand-reckoner, we place the digits next to one another in a horizontal row. The digit furthest to the right names the number of *ones*; the digit immediately to its left names the number of *tens*; the digit to its left names the number of *hundreds*; and so on. *Get the idea? The digit is the adjective and the* position (place) *of the digit names the power of ten.*

<u>*What the Digits Stand For:*</u>
1 stands for one (|)
2 stands for two (||)
3 stands for three (|||)
4 stands for four (||||)
5 stands for five (|||||)
6 stands for six (||||||)
7 stands for seven (|||||||)
8 stands for eight (||||||||)
9 stands for nine (|||||||||)
0 stands for none; and is explained in more detail in the text.

Example 7

In the place value numeral 253, what noun does 5 modify?

In other words, what place is held by the 5?

Answer: Tens

We have:

hundreds	tens	ones
2	5	3

That is, 253 is an abbreviation for

2 *hundreds*, 5 *tens*, and 3 *ones*.

Example 8

In the place value numeral, 532, what noun does 5 modify?

Answer: Hundreds

This time the placement of the digits

means:

hundreds	tens	ones
5	3	2

or: 5 *hundreds*, 3 *tens*, and 2 *ones*.

As an adjective, the digit 5 means the same thing in Example 7 as it does in Example 8. The difference is that in Example 7 we have 5 tens, while in Example 8 we have 5 *hundreds*.

We could say that a digit such as 5 has a face-value (which is always five) and a place-value (which depends on where we place the 5)

Note:

Both 253 and 532 consist of the same digits, but their placement gives us different numbers. This problem didn't happen with Roman numerals. Namely 253 would be written as CCXXXXXIII and 532 would be written as CCCCCXXXII. In this format we can see that we have five X's in one case and five C's in the other.

That is, the nouns are visible, taking such forms as I, X, C , and M. But in place value the nouns are determined solely from the placement of the digits.

The fact that the nouns (that is, the powers of ten) are known only by the placement of the digits causes a problem that didn't occur either with Roman numeral or the sand-reckoner. Namely:

Example 9

What number is named by CCIII?

Answer: Two hundred three (203)

CC means 2 *hundreds*

and III means 3 *ones*. So altogether we have 2 hundreds and 3 ones, which we read as two hundred (and) three.

The key point in Example 9 is that we do not need a symbol to tell us that there are *no* tens. That is, the symbol for ten is X, the *absence of X's tells us that we have no tens.*

In essence, we don't start counting until there is at least one thing to count. We don't walk into and empty room and say; "Look, there are no zebras!"

A similar thing happens with the sand-reckoner.

Example 10:

What number is named by:

Answer: Two hundred three

Each line stands for a different power of ten. In particular, we have:

hundreds tens ones

So we again have:

2 *hundreds* and 3 *ones*

In Example 10, we know there are no tens, because there are no markers on the tens-line. But in our modern place-value system, we need a special symbol to tell us when there are none of a particular denomination (power of ten).

Even though the lines look alike, we can tell them apart by their position. The key point is that the symbols that stand for the powers of ten are still visible.

To lead into this idea, let's return to Example 10 and imagine that we deleted the tens-line because there were no markers on it. What we'd see is:

(Figure 1)

Since we've already mentioned that the spacing between lines does not have to be uniform, Figure 1 would be interpreted as denoting twenty three.

That is, the line with two markers is the second from the left--and this means the tens-line

The point is that we do not erase a line that contains no markers. *But when the only way a power of ten can be seen is by the position of a digit, then the denomination disappears when a digit is absent.*

Now look at each row of the chart in Figure 2. As shown in the margin we can read each number as long as the denominations are labeled:

thousands	*hundreds*	*tens*	*ones*		
		2	3	→ → → →	*twenty three*
	2		3	→ → → →	*two hundred three*
	2	3		→ → → →	*two hundred thirty*
2			3	→ → → →	*two thousand three*
2		3		→ → → →	*two thousand thirty*
2	3			→ → → →	*two thousand three hundred*

(Figure 2)

But if the denominations are left out, each row consisists of a 2 followed by a 3 with various spacings. If we look at such configurations as: 2 3; 2   3 ; or 2   3; how can we tell what powers of ten are being described by the adjectives 2 and 3?

That is, how can we tell the <u>*place*</u> *of either the 2 or 3?*

The answer is that we introduce the notion of a *place-holder*. The numeral we use for the place-holder is 0. Most of us know that 0 is called zero and that it is used to indicate that we have none of a particular denomination.

Try not to confuse "zero" with "none". 0 is a place-holder and is as much a digit as is 1,2,3, and so on.

Example 11

What number is named by 203?

Answer: Two hundred three

Starting with the digit furthest to the right (in this case, 3) we locate the ones-place. As we move toward the left, 0 tells us to "hold the tens-place". The next digit to the left (namely, 2), therefore, holds the hundreds-place. So, in summary, we have:

In other words, we have no tens.

2 hundreds, no tens, and 3 ones.

The position of 0 tells us the "missing" denomination.

Example 12

What number is named by 230?

Answer: Two hundred thirty

We proceed as we did in the previous example. This time the digit furthest to the right is 0, which tells us that we have no ones. That is, 0 holds the ones-place. Moving to the left we then see that the 3 locates the tens-place and that the 2 locates the hundreds-place. So we have:

2 hundreds, 3 tens, and no ones

While the notion of a place-holder brought up questions that didn't exist with either Roman numerals or the sand-reckoner, the fact is that any problems it caused were more than offset by the ease and simplicity with which larger numbers could be expressed.

Example 13.

What number is named by 5730?

Answer: five thousand seven hundred thirty

Starting with the digit furthest to the right we have:

no ones, 3 tens, 7 hundreds, and 5 thousands.

Clearly, writing 5730 is simpler than having to write five thousand seven hundred thirty separate tally marks; and it is also a lot "neater" than writing MMMMMCCCCCCCXXX.

But while the level of sophistication may be different, the interesting point is that when numbers become sufficiently large, modern place-value numerals inherit the same problems that were present in the ancestors of place value. For example, how shall we read the number:

2734538904874 ?

2734538904874 is called a 13-digit numeral because it contains 13 digits. In counting digits we count 0's as well as repetitions. For example each of the three 4's is counted as a digit.

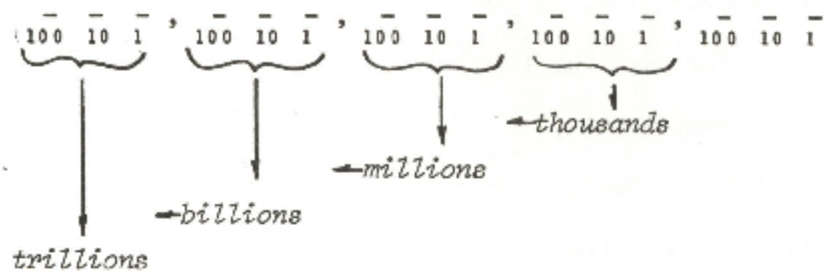
We'll answer this question in the next example but the idea behind the answer hinged on the fact that people felt that it was relatively easy to handle the numbers from 1 to 999. So what they attempted to do was to devise a system in which

this idea could be utilized. What they did was to start at the digit furthest to the right and counting from left to right, they placed a *comma* after every *third* digit. Applying this to 2734538904874, we'd get:

$$\underset{1 \leftarrow 321 \leftarrow 321 \leftarrow 321 \leftarrow 321}{2,734,538,904,874} \qquad (1)$$

If we start at the right and think of the commas as beginning a new number, th numbers we get are 2; 734; 538;904; and 874. In this way we never have more tha a 3-digit numeral to worry about.

Each cluster of digits modifies a noun. The name of these nouns are *thousands, millions, billions,* and *trillions*. Schematically, using dashes to stand for digits, we have:



A relatively easy way of learning the names of the denominations after "millions" is to think of the numerical prefixes "bi", "tri", "quad" (That is; 2, 3, 4,) etc. "bi-million" becomes "billion"; "tri-mi lion" becomes "trillion"; "quad-million" becomes "quadrillion" and so on.

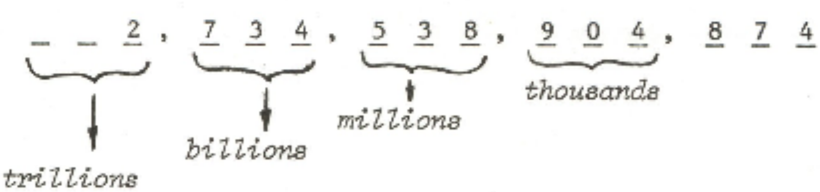
Let's try a few examples to make sure we have the main idea.

Example 14

How do we read the number:
2,734,538,904,874 ?

Answer: 2 trillion, 734 bi lion, 538 million, 904 thousand, 874

Using the diagram above, we have:



Notice that with "thousand "millions", "billions", an "trillions" as nouns, no numerical adjective has mo than three digits. That i we never need more than 99 of any denomination.

By the everyday standards of most of us, 13-digit numerals are probably more than we expect to encounter. So let's use smaller numbers in the next few examples.

Nevertheless, there are ma real-life cases in which even larger numbers are encountered. For that mat er, the national budget is in trillions of dollars.

Example 15

What number is named by 2,000,000,000?

Answer: 2 billion

In this case we have:

$$\underbrace{\underline{2}}_{\textit{billions}}\;,\;\underbrace{\underline{0}\;\underline{0}\;\underline{0}}_{\textit{millions}}\;,\;\underbrace{\underline{0}\;\underline{0}\;\underline{0}}_{\textit{thousands}}\;,\;\underbrace{\underline{0}\;\underline{0}\;\underline{0}}_{\textit{(units)}}$$

So that each adjective modifies a noun, we often refer to the pre-thousands as "units"

Example 15 illustrates a couple of interesting points. For one thing, in terms of thousands, millions, billions, trillions, and so on; our place-holder is 000 rather than 0. Secondly, when we use the word "billion" the numeral we see is 2. Somehow, to most people, "2 billion" is less awesome than "2,000,000,000".

In essence we're "trading-in" thousands so "none" means no hundreds, no tens and no ones.

Here, again, we see a good example of how the language of mathematics may cause more problems than the actual concepts.

In terms of the national budget, it seems like less money if we write \$1 trillion than if we write \$1,000,000,000,000. As another example there are 63,360 inches in 1 mile. Yet 1 mile seems easier to visualize than 63,360 inches.

As we shall see in Module 4 the proper choice of nouns can help us avoid having to deal with fractions!

Notice that in Example 15 it was important to keep track of the number as well as the placement of the 0's. Examples 16 and 17 further emphasize this idea.

Example 16

What number is named by 2,100,000?

Answer: 2 million 100 thousand

we have:

$$\underbrace{\underline{\ }\;\underline{\ }\;\underline{2}}_{\textit{millions}}\;,\;\underbrace{\underline{1}\;\underline{0}\;\underline{0}}_{\textit{thousands}}\;,\;\underline{0}\;\underline{0}\;\underline{0}$$

We sometimes write the answer entirely in words. For example, "Two million, one hundred thousand"

Example 17

What number is named by 2,010,000?

Answer: 2 million 10 thousand

This time we have:

$$\underbrace{\underset{100}{\underline{\ \ }}\ \underset{10}{\underline{\ \ }}\ \underset{1}{\underline{2}}}_{\text{millions}}\ ,\ \underbrace{\underset{100}{\underline{0}}\ \underset{10}{\underline{1}}\ \underset{1}{\underline{0}}}_{\text{thousands}}\ ,\ \underbrace{\underset{100}{\underline{0}}\ \underset{10}{\underline{0}}\ \underset{1}{\underline{0}}}_{\text{units}}$$

So we have 2 million and 10 thousand.

Notice that in the sequence 010, the 1 is in the tens-place. In 100, the 1 was in the hundreds-place.

The point is that the digits in both Examples 16 and 17 are a 2, a 1, and five 0's. Yet the placement of the 0's (as well as the other digits) affects the number named by the digits.

Always make sure that there are three digits between successive commas. Write 2,010,000 or 2,100,000 etc; but never write 2,1,000 or 2,10,000 or 2,01,000. The structure is that between successive commas we always have to express ones, tens, and hundreds.

Sometimes we are given the number in words and prefer to translate it into place-value language.

Example 18

Write three hundred eighty four as a place value numeral.

Answer: 384

Eighty means 8 tens. So we have 3 hundreds, 8 tens, and 4 ones; and we've already learned that this is written as 384.

Example 19

Write three hundred eighty four thousand as a place value numeral.

Answer: 384,000 (384 thousand)

"Three hundred eighty four" is still 384, but now 384 is modifying "thousands".

So we have 384 thousand, and in place value this is:

$$\underset{\textit{thousands}}{\underline{3}\ \underline{8}\ \underline{4}}\ ,\ \underset{\textit{units}}{\underline{0}\ \underline{0}\ \underline{0}}$$

Notice that the "dashes" actually hold the place for us. We use the 0's so that we may omit the dashes.

No matter how complicated the verbal description of the number becomes, we can break the problem down easily if we look for the key words "thousand(s)", "million(s)", "billion(s)" and so on.

Example 20

Write three hundred eighty four billion six hundred ninety million five hundred thirteen thousand two hundred thirty eight as a place value numeral.

Answer: 384,690,513,238

Don't let the words scare you. First locate the key words. Namely:

three hundred eighty four (384) *billion(s)*
six hundred ninety (690) *million(s)*
five hundred thirteen (513) *thousand(s)*
two hundred thirty eight (238) *(units)*

Notice how we never work with more than three digits at a time.

You should be able to read 1 through 999 in words. The rest is taken care of by the place value idea of "thousands", "millions" etc.

So we have:

$$\underbrace{\underline{3}\ \underline{8}\ \underline{4}}_{billions},\ \underbrace{\underline{6}\ \underline{9}\ \underline{0}}_{millions},\ \underbrace{\underline{5}\ \underline{1}\ \underline{3}}_{thousands},\ \underbrace{\underline{2}\ \underline{3}\ \underline{8}}_{(units)}$$

You should memorize the meaning of "thousands", "millions", "billions", "trillions" and so on, so that you know the order in which they occur. It is especially important to recognize when one of these denominations is missing.

Example 21

Write twenty five billion three hundred thousand as a place value numeral.

Answer: 25,000,300,000

We have twenty five (25) *billion;*
three hundred (300) *thousand.*

The denominations *"millions"* and *units"* are missing. So we use 000 to hold their places. That is, we write:

$\underline{\ \ }\ \underline{2}\ \underline{5}\ ,\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ ,\ \underline{3}\ \underline{0}\ \underline{0}\ ,\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }$

billions millions thousands

We may then omit the dashes by entering the appropriate place holders. We write the answer in its final form as:

25,000,300,000

If you merely "skip" the missing denominations the numeral might appear as: 25,300 which is twenty fiv thousand three hundred--a far cry from twenty five billion three hundred thou sand!

There is an old saying to the effect that the more things change the more they remain the same. In a way we have come a long way since our introduction of tally marks. Yet even with nouns like "millions", "billions", "trillions", and "quadrillions" it still seems complicated to keep track of the many place-values we need. For example, even with the use of commas it may not seem that easy to read the numeral

1,000,000,000,000,000,000,000,000

This number may seem pre-posterously large but it i roughly the number of atom in an ounce of water.

The scientist went on to invent the idea of *scientific notation* to name numerals that had the form of a 1 followed by only 0's. Applying scientific notation to 1,000,000,000,000,000,000,000,000 we invoke the following steps.

In subsequent modules we'l expand on the idea of scientific notation little by little.

Step 1. Simply write a 1.

So all we have so far is:

Step 2. Immediately to the right of the 1 place a 0.

Now we have 10

Step 3. Count the number of 0's and write this number, as a place value numeral, above and to the right of 0.

Since there are twenty-fou 0's in the given number, we'd write:

10^{24}

Before introducing further vocabulary let's try a few more examples.

Example 22

Write 10,000,000,000,000,000,000 in scientific notation.

Answer: 10^{19}

Since we have a 1 followed only by 0's we follow our three steps. First we write a 1. Then we follow it by a 0 to get 10. Then we count the number of 0's. There are nineteen 0's. So we write 19 above and to the right of 10 to get 10^{19}.

Example 23

Write 10,000,000,000,000,000,000,000 in scientific notation.

Answer: 10^{22}

In this example, we have a 1 followed by twenty-two 0's. So we write 10^{22}.

If we compare Examples 22 and 23 we see that keeping track of the number of 0's is analogous to keeping track of the difference between

||||||||||||||||||| and ||||||||||||||||||||||.

That is, whether it's nineteen tally marks or nineteen 0's, we still have to count them.

But when we write 10^{19} and 10^{22} we see at a glance that one of the numerals is a 1 followed by nineteen 0's and the other is a 1 followed by twenty two 0's.

Using scientific notation we'd write 1 as 10^0 because it is a 1 followed by no 0's. We'd write 10 as 10^1 because it is a 1 followed by one 0. We'd write 100 as 10^2 because it is a 1 followed by two 0's and so on.

There are other times when we're given the number in scientific notation and prefer to see it as a place value numeral.

Example 24

Write 10^{17} as a place value numeral.

Answer:
100,000,000,000,000,000

The 17 tells us that we have seventeen 0's.

So we start with a 1 and follow it by seventeen 0's to get:

$$\underset{}{1}\ \underset{1}{0}\ \underset{2}{0}\ \underset{3}{0}\ \underset{4}{0}\ \underset{5}{0}\ \underset{6}{0}\ \underset{7}{0}\ \underset{8}{0}\ \underset{9}{0}\ \underset{10}{0}\ \underset{11}{0}\ \underset{12}{0}\ \underset{13}{0}\ \underset{14}{0}\ \underset{15}{0}\ \underset{16}{0}\ \underset{17}{0}$$

We now start at the right and insert the commas in their proper places to get:

1 0 0,0 0 0,0 0 0,0 0 0,0 0 0,0 0 0

We may also read this as 100 quadrillion. That is, we have no units, no thousands, no millions, no billions, no trillions, and 100 quadrillions.

You may think of 10^{17} as being written as $1\,0^{17}$ to help remind you that we have a single 1 followed by 17 zeroes.

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General Summary of Scientific Notation:

If n denotes any counting number, then by

$$10^n$$

we mean the place value numeral that consists of a single 1 followed by n zeroes.

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Recall that a counting number is 1, 2, 3, 4 etc. 0 is not considered a counting number. However, we refer to 0,1,2,3 etc as the whole numbers. That is, 0 is the only whole number that is not a counting number.

We read "10^n" as "ten raised to the nth power" or more simply "ten to the nth" We call 10 the <u>*base*</u> *and n the exponent.*

In fact we refer to 10^n as the nth power of ten. *For example 10^3 is the 3rd power of ten. This ties in with our earlier remark that "ones", "tens", "hundreds", "thousands" and so on are called the powers of ten.*

If n is 0, then 10^n is 10^0 and means a single 1 followed by no 0's. That is 10^0 stands for 1.

This completes our development of place value enumeration. We have started with an idea as primitive as sign language and in a gradual progression we have evolved scientific notation. Such is the chronicle of human endeavor. We start with

the simplest solution to any problem we encounter. But solving one problem brings us to new plateaus of knowledge and produces new problems. To solve the new problem we look for a solution that grows out of our solution to the earlier problems. In this way progress is made, with each step requiring solutions to newer and bigger problems. Tally marks may look primitive when compared to place value. Yet place value itself looks primitive when compared to scientific notation.

And in the years ahead it is reasonable to assume that technology and other human needs will require a system of enumeration that makes scientific notation seem dull in comparison.

The trick is to study by going back to basic situations and adding on degrees of difficulty one step at a time.

This is the way we shall proceed in this text. We have started with tally marks and gone to scientific notation. In future modules we shall show how systems of enumeration led to methods of doing arithmetic. And by proceeding one small step at a time, the process should never become overwhelming.

In a similar way, algebra is a natural outgrowth of arithmetic. If one learns to understand arithmetic (as opposed simply to memorizing computational devices) then the study of algebra becomes much simpler--in fact it becomes a natural outgrowth of arithmetic.